Determining $|V_{ub}|$ from the $\overline{B} \to X_u \ell \overline{\nu}$ dilepton invariant mass spectrum[**]

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The invariant mass spectrum of the lepton pair in inclusive semileptonic $\overline{B} \to X_u \ell \overline{\nu}$ decay yields a model independent determination of $|V_{ub}|$ [1]. Unlike the lepton energy and hadronic invariant mass spectra, nonperturbative effects are only important in the resonance region, and play a parametrically suppressed role when $\mathrm{d}\Gamma/\mathrm{d}q^2$ is integrated over $q^2 > (m_B - m_D)^2$, which is required to eliminate the $\overline{B} \to X_c \ell \overline{\nu}$ background. We discuss these backgrounds for q^2 slightly below $(m_B - m_D)^2$, and point out that instead of $q^2 > (m_B - m_D)^2 = 11.6\,\mathrm{GeV}^2$, the cut can be lowered to $q^2 \gtrsim 10.5\,\mathrm{GeV}^2$. This is important experimentally, particularly when effects of a finite neutrino reconstruction resolution are included.

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A precise and model independent determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ub} is important for testing the Standard Model at B factories via the comparison of the angles and the sides of the unitarity triangle.

If it were not for the huge background from decays to charm, it would be straight-forward to determine $|V_{ub}|$ from inclusive semileptonic decays. Inclusive B decay rates can be computed model independently in a series in $\Lambda_{\mathrm{QCD}}/m_b$ and $\alpha_s(m_b)$ using an operator product expansion (OPE) [2,3,4,5], and the result may schematically be written as

$$ d\Gamma = \begin{pmatrix} b \text{ quark} \\ \text{decay} \end{pmatrix} \times \left\{ 1 + \frac{0}{m_b} + \frac{f(\lambda_1, \lambda_2)}{m_b^2} + \ldots + \frac{\alpha_s}{\pi}(\ldots) + \frac{\alpha_s^2}{\pi^2}(\ldots) + \ldots \right\}. \quad (1) $$

At leading order, the B meson decay rate is equal to the b quark decay rate. The leading nonperturbative corrections of order $\Lambda_{\mathrm{QCD}}^2/m_b^2$ are characterized by two heavy quark effective theory (HQET) matrix elements, usually called λ_1 and λ_2. These matrix elements also occur in the expansion of the B and B^* masses in powers of $\Lambda_{\mathrm{QCD}}/m_b$,

$$ m_B = m_b + \overline{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b} + \ldots, \qquad m_{B^*} = m_b + \overline{\Lambda} - \frac{\lambda_1 - \lambda_2}{2m_b} + \ldots. \quad (2) $$

Similar formulae hold for the D and D^* masses. The parameters $\overline{\Lambda}$ and λ_1 are independent of the heavy b quark mass, while there is a weak logarithmic scale dependence in λ_2. The measured $B^* - B$ mass splitting fixes $\lambda_2(m_b) = 0.12 \, \mathrm{GeV}^2$, while $\overline{\Lambda}$ and λ_1 (or, equivalently, a short distance b quark mass and λ_1) may be determined from other physical quantities [6,7,8]. Thus, a measurement of the total $B \to X_u \ell \overline{\nu}$ rate would provide a $\sim 5\%$ determination of $|V_{ub}|$ [9,10].

Unfortunately, the $\overline{B} \to X_u \ell \overline{\nu}$ rate can only be measured imposing cuts on the phase space to eliminate the ~ 100 times larger $\overline{B} \to X_c \ell \overline{\nu}$ background. Since the predictions of the OPE are only model independent for *sufficiently inclusive* observables, these cuts can destroy the convergence of the expansion. This is the case for two kinematic regions for which the charm background is absent and which have received much attention: the large lepton energy region, $E_\ell > (m_B^2 - m_D^2)/2m_B$, and the small hadronic invariant mass region, $m_X < m_D$ [11,12,13].

The poor behaviour of the OPE for these quantities is slightly subtle, because in both cases there is sufficient phase space for many different resonances to be produced in the final state, so an inclusive description of the decays is still appropriate. However, in both of these regions of phase space the $\overline{B} \to X_u \ell \overline{\nu}$ decay products are dominated by high energy, low invariant mass hadronic states,

$$ E_X \sim m_b, \ m_X^2 \sim \Lambda_{\mathrm{QCD}} m_b \gg \Lambda_{\mathrm{QCD}}^2 \quad (3) $$

(where E_X and m_X are the energy and invariant mass of the final hadronic state). In this region the differential rate is very sensitive to the details of the wave function of

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the b quark in the B meson. Since the OPE is just sensitive to local matrix elements corresponding to expectation values of operators in the meson, the first few orders in the OPE do not contain enough information to describe the decay, and as a result the OPE does not converge.

This is simple to see by considering the kinematics. A b quark in a B meson has momentum

$$p_b^\mu = m_b v^\mu + k^\mu \tag{4}$$

where v^μ is the four-velocity of the quark, and k^μ is a small residual momentum of order $\Lambda_{\rm QCD}$. If the hadron decays to leptons with momentum q and light hadrons with total momentum p_X, the invariant mass of the light hadrons may be written

$$m_X^2 = (m_b v + k - q)^2 = (m_b v - q)^2 + 2k \cdot (m_b v - q) + O(\Lambda_{\rm QCD}^2). \tag{5}$$

The first term in the expansion is $O(m_b^2)$ over most of phase space, while the second is $O(\Lambda_{\rm QCD} m_b)$, and so is suppressed over most of phase space. The OPE presumes that this power counting holds, so that the second term may be treated as a small perturbation. However, if E_X is large and m_X is small, $m_b v - q$ is almost light-like,

$$m_b v^\mu - q^\mu = (E_X, 0, 0, E_X) + O(\Lambda_{\rm QCD}) \tag{6}$$

in the b rest frame where $v^\mu = (1, 0, 0, 0)$. Since $E_X \sim O(m_b)$, $(m_b v - q)^2 = O(\Lambda_{\rm QCD} m_b)$. Thus, in this region the first two terms in (5) are of the same order (but still parametrically larger than the remaining terms), and the invariant mass of the final hadronic state reflects the distribution of the light-cone component of the residual momentum of the heavy quark in the hadron,

$$m_X^2 = (m_b v - q)^2 + 2E_X k_+ + \dots, \quad k_+ \equiv k_0 + k_3. \tag{7}$$

Since the differential rate in this region depends on the invariant mass of the final state, it is therefore sensitive at leading order to the light-cone wave function of the heavy quark in the meson, $f(k_+)$.

In terms of the OPE, this light-cone wave function arises because of subleading terms in the OPE proportional to $E_X \Lambda_{\rm QCD}/m_X^2$, which are suppressed over most of phase space but are $O(1)$ in the region (3). It has been shown that the most singular terms in the OPE may be resummed into a nonlocal operator whose matrix element in a B meson is the light-cone structure function of the meson. Since $f(k_+)$ is a nonperturbative function, it cannot be calculated analytically, so the rate in the region (3) is model-dependent even at leading order in $\Lambda_{\rm QCD}/m_b$.

The situation is illustrated in Fig. 1, where we have plotted the lepton energy and hadronic invariant mass spectra in the parton model (dashed curves) and incorporating a simple one-parameter model for the distribution function (solid curves) [17]

$$f(k_+) = \frac{32}{\pi^2 \Lambda} (1-x)^2 \exp\left[-\frac{4}{\pi}(1-x)^2\right] \Theta(1-x), \qquad x \equiv \frac{k_+}{\Lambda}, \qquad \Lambda = 0.48\,{\rm GeV}. \tag{8}$$
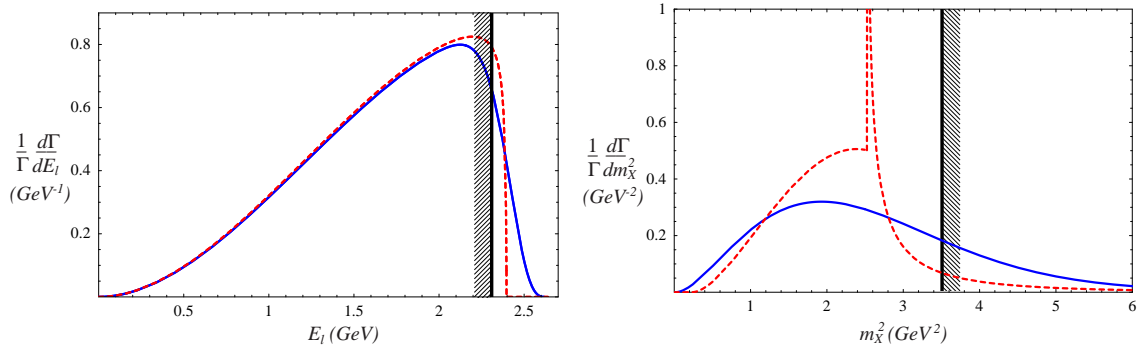
Figure 1: The shapes of the lepton energy and hadronic invariant mass spectra. The dashed curves are the b quark decay results to $\mathcal{O}(\alpha_s)$, while the solid curves are obtained by smearing with the model distribution function $f(k_+)$ in Eq. (8). The unshaded side of the vertical lines indicate the region free from charm background.

The differences between the curves in the regions of interest indicate the sensitivity of the spectrum to the precise form of $f(k_+)$. Currently, there are measurements of $|V_{ub}|$ from both methods. From the lepton energy cut, the PDG reports $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$, while a recent DELPHI measurement using the hadronic invariant mass cut gives $|V_{ub}/V_{cb}| = 0.103^{+0.011}_{-0.012}\,(\text{syst.}) \pm 0.016\,(\text{stat.}) \pm 0.010\,(\text{theory})$ [16]. In both cases, the theoretical error is an estimate based on varying different models of $f(k_+)$, and so these measurements are no more model-independent than the exclusive measurement from $B \to \rho\ell\overline{\nu}$. While it may be possible in the future to extract $f(k_+)$ from the $B \to X_s\gamma$ photon spectrum [14,18], unknown order Λ_{QCD}/m_b corrections arise when relating this to semileptonic $b \to u$ decay, limiting the accuracy with which $|V_{ub}|$ may be obtained.

Clearly, one would like to be able to find a cut which eliminates the charm background but does not destroy the convergence of the OPE, so that the distribution function $f(k_+)$ is not required. In Ref. [1] we pointed out that this is the situation for a cut on the dilepton invariant mass. Decays with $q^2 \equiv (p_\ell + p_{\overline{\nu}})^2 > (m_B - m_D)^2$ must arise from $b \to u$ transition. Such a cut forbids the hadronic final state from moving fast in the B rest frame, and simultaneously imposes $m_X < m_D$ and $E_X < m_D$. Thus, the light-cone expansion which gives rise to the shape function is not relevant in this region of phase space [13,19]. The effect of smearing the q^2 spectrum with the model distribution function in Eq. (8) is illustrated in Fig. 2. It is clearly a subleading effect. The Dalitz plots relevant for the charged lepton energy and hadronic invariant mass cuts are shown in Fig. 3. Note that the region selected by a q^2 cut is entirely contained within the m_X^2 cut, but because the dangerous region of high energy, low invariant mass final states is not included with the q^2 cut, the OPE does not break down. It is also important to note, however, that the q^2 cut does make the OPE worse than for the full rate; as we will show, the relative size of the unknown $\Lambda_{\text{QCD}}^3/m_b^3$ terms grows
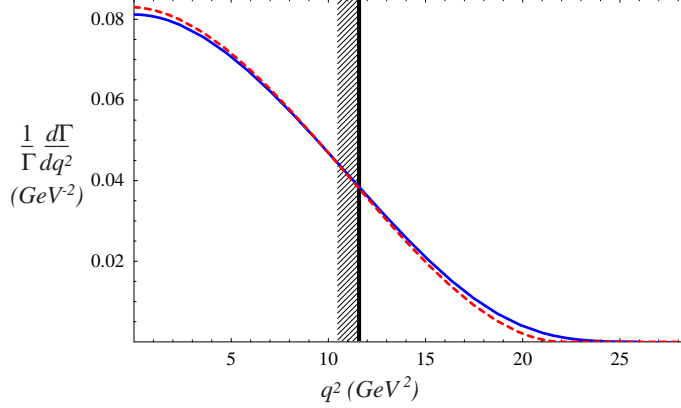
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Figure 2: The dilepton invariant mass spectrum. The notation is the same as in Fig. 1.

as the q^2 cut is raised. Equivalently, as was stressed in [20], the effective expansion parameter for this region is $\Lambda_{\rm QCD}/m_c$, not $\Lambda_{\rm QCD}/m_b$.

The $\overline{B} \to X_u\ell\overline{\nu}$ decay rate with lepton invariant mass above a given cutoff can therefore be reliably computed working to a fixed order in the OPE (i.e., ignoring the light-cone distribution function),

$$
\begin{aligned}
\frac{1}{\Gamma_0} \frac{{\rm d}\Gamma}{{\rm d}\hat{q}^2} &= \left(1 + \frac{\lambda_1}{2m_b^2}\right) 2\,(1 - \hat{q}^2)^2\,(1 + 2\hat{q}^2) + \frac{\lambda_2}{m_b^2}\,(3 - 45\hat{q}^4 + 30\hat{q}^6) \\
&\quad + \frac{\alpha_s(m_b)}{\pi}\,X(\hat{q}^2) + \left(\frac{\alpha_s(m_b)}{\pi}\right)^2 \beta_0\,Y(\hat{q}^2) + \dots,
\end{aligned} \tag{9}
$$

where $\hat{q}^2 = q^2/m_b^2$, $\beta_0 = 11 - 2n_f/3$, and $\Gamma_0 = G_F^2\,|V_{ub}|^2\,m_b^5/(192\,\pi^3)$ is the tree level $b \to u$ decay rate. The ellipses in Eq. (9) denote terms of order $(\Lambda_{\rm QCD}/m_b)^3$ and order α_s^2 terms not enhanced by β_0. The function $X(\hat{q}^2)$ is known analytically [21], whereas $Y(\hat{q}^2)$ was computed numerically [22]. The order $1/m_b^3$ nonperturbative corrections are also known [23], as are the leading logarithmic perturbative corrections proportional to $\alpha_s^n \log^n(m_c/m_b)$ [20]. The matrix element of the kinetic energy operator, λ_1, only enters the $\hat{q}^2$ spectrum in a very simple form, because the unit operator and the kinetic energy operator are related by reparameterization invariance [24].

The relation between the total $\overline{B} \to X_u\ell\overline{\nu}$ decay rate and $|V_{ub}|$ is known at the $\sim 5\%$ level [9,10],

$$
|V_{ub}| = (3.04 \pm 0.06 \pm 0.08) \times 10^{-3} \left(\frac{\mathcal{B}(\overline{B} \to X_u\ell\overline{\nu})|_{q^2 > q_0^2}}{0.001 \times F(q_0^2)} \frac{1.6\,{\rm ps}}{\tau_B}\right)^{1/2}, \tag{10}
$$

where $F(q_0^2)$ is the fraction of $\overline{B} \to X_u\ell\overline{\nu}$ events with $q^2 > q_0^2$, satisfying $F(0) = 1$. The errors explicitly shown in Eq. (10) are the estimates of the perturbative and nonperturbative uncertainties in the upsilon expansion [9] respectively. At the
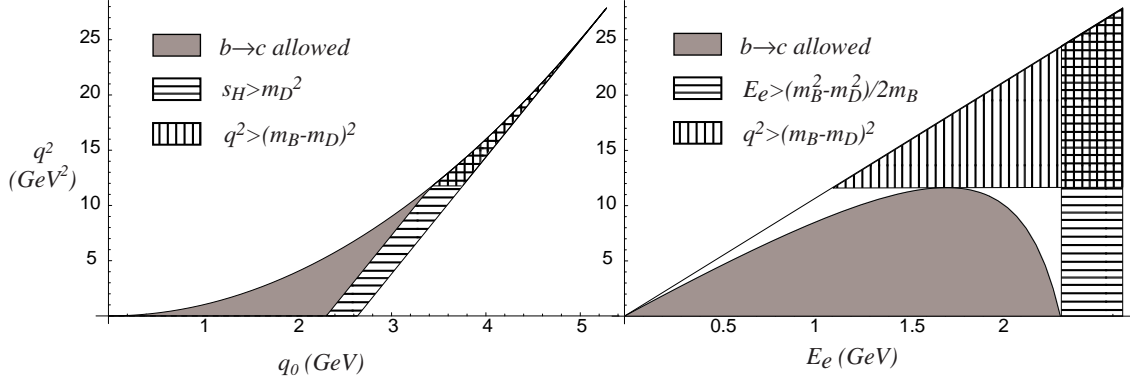
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Figure 3: Dalitz plots relevant for $\overline{B} \to X_u \ell \overline{\nu}_\ell$. The shaded regions indicate the part of phase space where $\overline{B} \to X_c \ell \overline{\nu}_\ell$ background is present, and the vertical dashed regions corresponds to the cut $q^2 > (m_B - m_D)^2$. In the $q^2 - q_0$ plane, the horizontal dashed region corresponds to an invariant mass cut $m_X^2 > m_D^2$, whereas in the $q^2 - E_\ell$ plane the horizontal dashed region corresponds to the charged lepton energy cut $E_\ell > (m_B^2 - m_D^2)/2m_B$. Note that at tree level, $b \to u$ semileptonic decay populates the entire triangle on the right-hand plot, but only the right boundary of the left-hand plot.

present time the biggest uncertainty is due to the error of a short distance b quark mass, whichever way it is defined [20]. (This can be cast into an uncertainty in an appropriately defined $\overline{\Lambda}$, or the nonperturbative contribution to the $\Upsilon(1S)$ mass, etc.) By the time the q^2 spectrum in $\overline{B} \to X_u \ell \overline{\nu}$ is measured, this uncertainty should be reduced from extracting m_b from the hadron mass [6] or lepton energy [7] spectra in $\overline{B} \to X_c \ell \overline{\nu}$, or from the photon energy spectrum [8] in $B \to X_s \gamma$. The uncertainty in the perturbation theory calculation will be largely reduced by computing the full order α_s^2 correction in Eq. (10). The largest "irreducible" uncertainty is from order $\Lambda_{\rm QCD}^3/m_b^3$ terms in the OPE, the estimated size of which is shown in Fig. 4, together with our central value for $F(q_0^2)$, as functions of q_0^2.

There is another advantage of the q^2 spectrum over the m_X spectrum to measure $|V_{ub}|$. In the variable m_X, about 20% of the charm background is located right next to the $b \to u$ "signal region", $m_X < m_D$, namely $\overline{B} \to D\ell\overline{\nu}$ at $m_X = m_D$. In the variable q^2, the charm background just below $q^2 = (m_B - m_D)^2$ comes from the lowest mass X_c states. Their q^2 distributions are well understood based on heavy quark symmetry [25], since this region corresponds to near zero recoil. Fig. 5 shows the $\overline{B} \to D\ell\overline{\nu}$ and $\overline{B} \to D^*\ell\overline{\nu}$ decay rates using the measured form factors [26] (and $|V_{ub}| = 0.0035$). The $\overline{B} \to X_u\ell\overline{\nu}$ rate is the flat curve. Integrated over the region $q^2 > (m_B - m_{D^*})^2 \simeq 10.7 \, {\rm GeV}^2$, the uncertainty of the $B \to D$ background is small due to its $(w^2-1)^{3/2}$ suppression compared to the $\overline{B} \to X_u\ell\overline{\nu}$ signal. This uncertainty will be further reduced in the near future. This increases the $b \to u$ region relevant
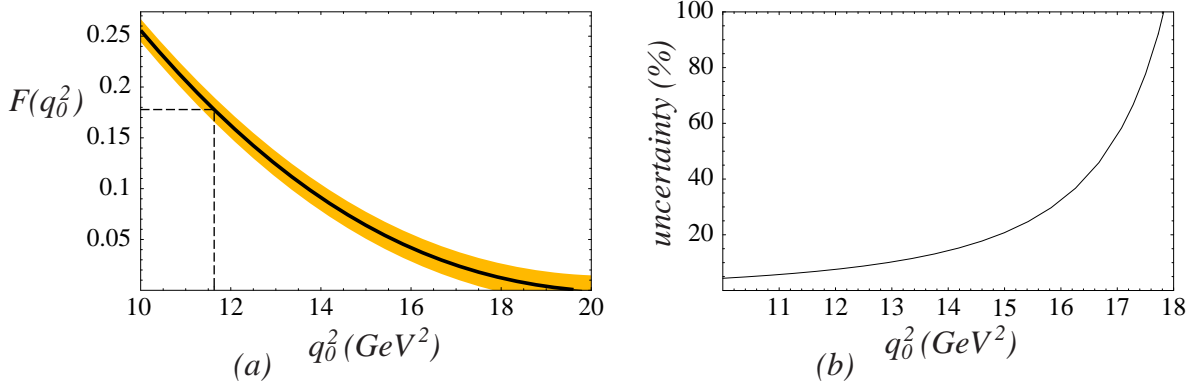
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Figure 4: (a) The fraction of $\overline{B} \to X_u \ell \overline{\nu}$ events with $q^2 > q_0^2$, $F(q_0^2)$, in the upsilon expansion. The dashed line indicates the lower cut $q_0^2 = (m_B - m_D)^2 \simeq 11.6 \, \text{GeV}^2$, which corresponds to $F = 0.178 \pm 0.012$. The shaded region is the estimated uncertainty due to $\Lambda_{\text{QCD}}^3 / m_b^3$ terms; which is shown in (b) as a percentage of $F(q_0^2)$.
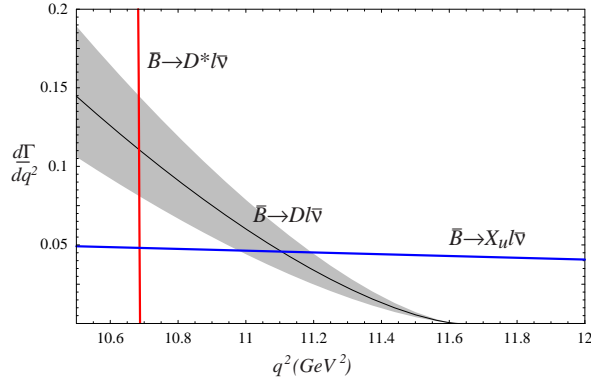

Figure 5: Charm backgrounds near $q^2 = (m_B - m_D)^2$ (arbitrary units). The shaded region denotes the uncertainty on the $\overline{B} \to D\ell\overline{\nu}$ rate.

for measuring $|V_{ub}|$ by $\sim 1 \, \text{GeV}^2$. The $B \to D^*$ rate is only suppressed by $(w^2 - 1)^{1/2}$ near zero recoil, and therefore it is more difficult to subtract it reliably from the $b \to u$ signal. The nonresonant $D\pi$ final state contributes in the same region as $\overline{B} \to D^*$, and it is reliably predicted to be small near maximal q^2 (zero recoil) based on chiral perturbation theory [27]. The D^{**} states only contribute for $q^2 < 9 \, \text{GeV}^2$, and some aspects of their q^2 spectra are also known model independently [28].

Concerning experimental considerations, measuring the q^2 spectrum requires reconstruction of the neutrino four-momentum, just like measuring the hadronic invariant mass spectrum. A lepton energy cut may be required for this technique, however, the constraint $q^2 > (m_B - m_D)^2$ automatically implies $E_\ell > (m_B - m_D)^2 / 2m_B \simeq 1.1 \, \text{GeV}$ in the B rest frame. Even if the E_ℓ cut has to be slightly larger than this, the utility of our method will not be affected, but a calculation including the effects

of arbitrary E_ℓ and q^2 cuts would be required. If experimental resolution on the reconstruction of the neutrino momentum necessitates a significantly larger cut than $q_0^2 = (m_B - m_D)^2$, then the uncertainties in the OPE calculation of $F(q_0^2)$ increase. In this case, it may be possible to obtain useful model independent information on the q^2 spectrum in the region $q^2 > m_{\psi(2S)}^2 \simeq 13.6\,\mathrm{GeV}^2$ from the q^2 spectrum in the rare decay $\overline{B} \to X_s \ell^+ \ell^-$, which may be measured in the upcoming Tevatron Run-II.

In conclusion, we have shown that the q^2 spectrum in inclusive semileptonic $\overline{B} \to X_u \ell \overline{\nu}$ decay gives a model independent determination of $|V_{ub}|$ with small theoretical uncertainty. Nonperturbative effects are only important in the resonance region, and play a parametrically suppressed role when $\mathrm{d}\Gamma/\mathrm{d}q^2$ is integrated over $q^2 > (m_B - m_D)^2$, which is required to eliminate the charm background. This is a qualitatively better situation than other extractions of $|V_{ub}|$ from inclusive charmless semileptonic B decay.

Acknowledgements

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